STAT537: Statistics for Research I: HW#3

Due on Sep. 13, 2016

Dr. Schmidhammer TR 11:10am – 12:25pm

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Contents

Problem 1

Let Y be a random variable having a binomial distribution with parameters n=20

Solution. 1. **The exact binomial distribution:**

- When $p = 0.1$, $P[Y \leq 1] = 0.391747$
- When $p = 0.3$, $P[Y \leq 1] = 0.00763726$

2. **The normal approximation to the binomial without the continuity correction:**

- When $p = 0.1$, $Y \leq 1$, the mean of this binomial distribution is given by

$$\mu = np = 20 \cdot 0.1 = 2$$

while the standard deviation is given by

$$\sigma = \sqrt{np(1-p)} = \sqrt{20 \cdot 0.1 \cdot 0.9} = 1.341641$$

$$P[Y \leq 1] = P\left(z < \frac{y-\mu}{\sigma}\right) = 0.2280283$$

- Similarly, when $p = 0.3$, $P[Y \leq 1] = 0.007348711$

3. **The normal approximation to the binomial with the continuity correction:**

- When $p = 0.1$, $P[Y \leq 1]$ the mean of this binomial distribution is given by

$$\mu = np = 20 \cdot 0.1 = 2$$

while the standard deviation is given by

$$\sigma = \sqrt{np(1-p)} = \sqrt{20 \cdot 0.1 \cdot 0.9} = 1.341641$$

$$P[Y \leq 1] = P[Y < 1.5] \approx P\left(z < \frac{y+0.5-\mu}{\sigma}\right) = 0.3546941$$

- Similarly, when $p = 0.3$, $P[Y \leq 1] = 0.01405402$

4. **Does the continuity correction always yield a better approximation?** From the results from part 2 and part 3, we can conclude that the continuity correction not always yield a better approximation, it same that continuity correction gives a better result when the probability is small, while without continuity correction gives a better result when the probability is larger.

□

Problem 2

Ott 4.54

Solution. (a) $P = 0.1973982$

(b) $P = 0.3849303$

□

Problem 3

Ott 4.64

Solution. Since $\mu = 100$, and $\sigma = 8$, then

$$z = \frac{y-\mu}{\sigma} = \frac{y-100}{8}$$

(a) If $y = 100$, then $z = \frac{y-100}{8} = \frac{100-100}{8} = 0$

$$P(y > 100) = 1 - p(y \leq 100) = 1 - P(z) = 0.5$$

(b) If $y = 105$, then $z = \frac{y-100}{8} = \frac{105-100}{8} = \frac{5}{8}$

$$P(y > 105) = 1 - p(y \leq 105) = 1 - P(z) = 0.2659855$$

(c) If $y = 110$, then $z = \frac{y-100}{8} = \frac{110-100}{8} = \frac{10}{8}$

$$P(y < 100) = P(z) = 0.8943502$$

(d) If $y_1 = 120, y_2 = 88$, then $z_1 = \frac{120-100}{8} = \frac{20}{8}, z_2 = \frac{88-100}{8} = -\frac{12}{8}$,

$$P(88 < y < 100) = P(z_2 < z < z_1) = 0.9269831$$

(e) If $y_1 = 108, y_2 = 100$, then $z_1 = \frac{108-100}{8} = 1, z_2 = \frac{100-100}{8} = 0$,

$$P(88 < y < 100) = P(z_2 < z < z_1) = 0.3413447$$

□

Appendix

R code for HW#3

Listing 1: R Source code for HW#3

```
# The exact binomial distribution
prole11 = pbinom(1, size=20, prob=0.1)   # P[X<=1]
prole12 = pbinom(1, size=20, prob=0.3)   # P[X<=1]

prole11
prole12

# The normal approximation to the binomial without the continuity correction
p=0.3; n=20; y=1
mu = n*p
sigma = sqrt(n*p*(1-p))
sigma
# P(Y<= 1)=P(z<(y-mu)/sigma)
z = (y-mu)/sigma
p = pnorm(z)
p
```

```
# The normal approximation to the binomial with the continuity correction

p=0.3; n=20; y=1
mu = n*p
sigma = sqrt(n*p*(1-p))
sigma
# P(Y<= 1)=P(z<(y-mu)/sigma)
z = (y+0.5-mu)/sigma
p = pnorm(z)
p

##############################################
# Ott 4.54
z1 = 1.7; z2 =0.7
p1= pnorm(z1)-pnorm(z2)
p1

z1 = 0; z2 =-1.2
p2= pnorm(z1)-pnorm(z2)
p2

##############################################
# Ott 4.64
y=100; mu = 100;sigma =8
# P(Y<= 1)=P(z<(y-mu)/sigma)
z = (y-mu)/sigma
p1 = 1-pnorm(z)
p1

y=105; mu = 100;sigma =8
# P(Y<= 1)=P(z<(y-mu)/sigma)
z = (y-mu)/sigma
p2 = 1-pnorm(z)
p2

y=110; mu = 100;sigma =8
# P(Y<= 1)=P(z<(y-mu)/sigma)
z = (y-mu)/sigma
p3 = pnorm(z)
p3


y1=120; y2=88; mu = 100;sigma =8
# P(Y<= 1)=P(z<(y-mu)/sigma)
z1 = (y1-mu)/sigma
z2 = (y2-mu)/sigma
p4 = pnorm(z1)-pnorm(z2)
p4

y1=108; y2=100; mu = 100;sigma =8
# P(Y<= 1)=P(z<(y-mu)/sigma)
z1 = (y1-mu)/sigma
z2 = (y2-mu)/sigma
```

```
p5 = pnorm(z1)-pnorm(z2)
p5
```